thermal conductivity tensor components; e, charge on an electron; $\rho^{(n)}$, carrier concentration of the n-th extremum; $\langle\tau_{ii}^{n}\rangle$, components of the average relaxation time of the n-th extremum; $m_{ii}^{(n)}$, tensor components of the effective carrier mass of the n-th extremum; $\alpha^{(n)}$, isotropic thermal emf due to carriers of the n-th extremum; $\varkappa_{ii}^{(0)}$, lattice component of the thermal conductivity; T, absolute temperature; $\langle\tau_{ii}^{(n)F}\rangle$ average carrier relaxation time for scattering by acoustic phonons; $\langle\tau_{ii}^{(n)I}\rangle$, average relaxation time for scattering by ionized impurities; k_0, Boltzmann constant; E, energy.

LITERATURE CITED

1. V. I. Chervyakova, Thermoelectric Instruments [in Russian], Gosénergoizdat, Moscow – Leningrad (1963).
2. C. A. Hogarth (editor), Materials Used in Semiconductor Devices, Wiley (1965).
3. F. J. Wilkins and T. A. Deacon, Proc. IEE, 112, No. 4, 794 (1965).
4. USA Patent No. 3,533,855.
5. B. M. Gol'tzman, V. A. Kudinov, and I. A. Smirnov, Semiconductor Thermoelectric Materials Based on Bi_2Te_3 [in Russian] (edited by B. Ya. Moizhes), Nauka, Moscow (1972).
6. V. S. Popov, Élektrichestvo, No. 9 (1958).
7. K. D. Tovstyuk, D. M. Bercha, Z. V. Pankevich, and I. M. Rarenko, Phys. Status Solidi, 13, 207 (1966).
8. L. I. Anatychuk and A. M. Gnatyuk, Izv. Akad. Nauk SSSR, Neorgan. Mater., No. 8 (1969).
9. I. P. Grinberg and E. E. Lashchuk, Control-Measurement Techniques [in Russian], No. 8 (1969).
10. V. S. Popov, Electrothermal Converters in Computer Engineering [in Russian], Tekhnika, Kiev (1971).
11. T. B. Rozhdestvenskaya, Electrical Comparators for Accurate Current, Voltage, and Power Measurements [in Russian], Standartgiz, Moscow (1964).

SOLUTION OF A PROBLEM ABOUT EVAPORATION OF SPHERICAL METAL PARTICLES IN AN ARC FLAME BY AN INTEGRAL EQUATIONS METHOD

É. A. Arinshtein and A. A. Kislitsyn

UDC 536.42

The problem about the evaporation of metal particles in a plasma is reduced to a single-phase nonstationary Stefan problem, whose solution is obtained by an integral equations method. Computations are performed for spherical lead particles with an initial radius of $R = 7\cdot10^{-3}$ cm.

Plasma interaction with solid particles is of great interest in plasma physics. The processes occuring here can be modeled by the following problem.

A globular metal particle with initial radius R_0 enters a plasma whereupon it starts to evaporate. Our problem is to find the law of the time change in the particle radius $r_1(t)$. Experiments on the evaporation of lead and tin particles are elucidated in [1].

The problem under consideration is treated in this paper as a Stefan problem and its theoretical solution by an integral equations method is proposed [2, 3]. The temperature at any point of the particle can be found from the expression

$$T(r,\ t)=\int_0^t T(\rho,\ \tau)\,G|_{\rho=r_1(\tau)}\frac{dr_1}{d\tau}d\tau+a\int_0^t\left(G\frac{\partial T(\rho,\ \tau)}{\partial\rho}-T(\rho,\ \tau)\frac{\partial G}{\partial\rho}\right)\bigg|_{\rho=r_1(\tau)}d\tau, \tag{1}$$

Tyumen' State University. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 31, No. 2, pp. 306-310, August, 1976. Original article submitted March 17, 1975.

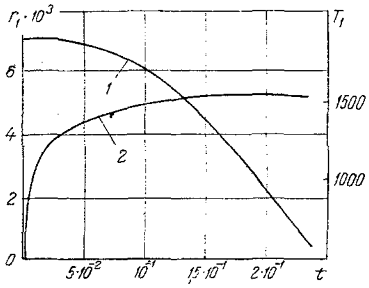

Fig. 1. Dynamics of the evaporation process for a spherical lead particle in an arc flame: 1) time dependence of the radius of the evaporating particle; 2) temperature of the evaporation surface $T_1(t)$.

where the Green's function G is

$$G(r, \rho, t, \tau) = \frac{r}{2\rho\sqrt{\pi a(t-\tau)}}\left\{\exp\left[-\frac{(r-\rho)^2}{4a(t-\tau)}\right] - \exp\left[\frac{(r+\rho)^2}{4a(t-\tau)}\right]\right\}. \tag{2}$$

For brevity, we set $T(r, 0) \equiv 0$ in writing (1). The motion of the surface is determined by the equation of evaporation kinetics [4, 5]

$$\left|\frac{dr_1(t)}{dt}\right| = v(t) = v_0 \exp\left\{-\frac{L\mu}{RT_1(t)}\right\}, \tag{3}$$

where R is the gas constant; $v_0 = \bar{c}(3/4\pi)^{1/3}$, $\bar{c}$ is the mean speed of sound in the metal. The energy conservation law, which can be written as

$$-\lambda \frac{\partial T}{\partial r}\bigg|_{r=r_1(t)} = q(t) + \gamma \frac{dr_1}{dt} L, \tag{4}$$

is satisfied on the surface in addition to the kinetics equation. Let us consider the heat flux to be determined by the expression

$$q(t) = \alpha[T_p - T_1(t)], \tag{5}$$

where T_p is the plasma temperature.

Setting $r = r_1$ in (1) and eliminating the functions $\partial T/\partial r$ and dr_1/dt by using (3) and (4), we obtain an integral equation in the temperature $T_1(t)$ of the evaporation surface:

$$T_1(t) = -\int_0^t T_1(\tau) G\bigg|_{r=\rho=r_1(\tau)} v_0 \exp\left[-\frac{L\mu}{RT_1(\tau)}\right] d\tau + \int_0^t \left\{G\bigg|_{r=\rho=r_1(\tau)}\left[\frac{\gamma v_0 L}{\lambda}\exp\left(-\frac{L\mu}{RT_1(\tau)}\right) - \frac{\alpha}{\lambda}(T_p - T_1(\tau))\right] - T_1(\tau)\frac{\partial G}{\partial \rho}\bigg|_{r=\rho=r_1(\tau)}\right\} d\tau \tag{6}$$

which can be solved by successive approximations by using an electronic computer. Expanding the expressions for G and $\partial G/\partial \rho$, we write (6) as

$$T_1(t) = \frac{1}{2\sqrt{\pi a}} \sum_{n=1}^{7} I_n(t), \tag{7}$$

where

$$I_1 = v_0 \int_0^t T_1(\tau) \exp\left[-\frac{L\mu}{RT_1(\tau)}\right]\left\{\exp\left[-\frac{r_1^2(\tau)}{a(t-\tau)}\right] - 1\right\}\frac{d\tau}{\sqrt{t-\tau}}, \tag{8}$$

TABLE 1. Values of the Integrals I_n at Different Times

Integral	$t=0,04$ sec	$t=0,1$ sec	$t=0,2$ sec
I_1	$-2,4\cdot10^{-2}$	$-1,7\cdot10^{-1}$	$-1,0\cdot10^{-1}$
I_2	5,96	$5,56\cdot10^{1}$	$2,02\cdot10^{2}$
I_3	$2,02\cdot10^{3}$	$2,09\cdot10^{3}$	$2,03\cdot10^{3}$
I_4	5,76	$5,46\cdot10^{1}$	$2,01\cdot10^{2}$
I_5	$-3,3\cdot10^{1}$	$-3,3\cdot10^{1}$	$-1,0\cdot10^{1}$
I_6	$2,7\cdot10^{2}$	$3,2\cdot10^{2}$	$1,7\cdot10^{2}$
I_7	1,09	1,23	$6,8\cdot10^{-1}$

$$I_2=\frac{\mu v_0 L}{3R}\int_0^t \exp\left[-\frac{L\mu}{RT_1(\tau)}\right]\frac{d\tau}{\sqrt{t-\tau}}, \tag{9}$$

$$I_3=\frac{a\alpha T_p}{\lambda}\int_0^t\left\{1-\exp\left[-\frac{r_1^2(\tau)}{a(t-\tau)}\right]\right\}\frac{d\tau}{\sqrt{t-\tau}}, \tag{10}$$

$$I_4=\frac{\mu v_0 L}{3R}\int_0^t \exp\left[-\frac{r_1^2(\tau)}{a(t-\tau)}\right]\exp\left[-\frac{L\mu}{RT_1(\tau)}\right]\frac{d\tau}{\sqrt{t-\tau}}, \tag{11}$$

$$I_5=\frac{a\alpha}{\lambda}\int_0^t T_1(\tau)\left\{\exp\left[-\frac{r_1^2(\tau)}{a(t-\tau)}\right]-1\right\}\frac{d\tau}{\sqrt{t-\tau}}, \tag{12}$$

$$I_6=a\int_0^t\frac{T_1(\tau)}{r_1(\tau)}\left\{1-\exp\left[-\frac{r_1^2(\tau)}{a(t-\tau)}\right]\right\}\frac{d\tau}{\sqrt{t-\tau}}, \tag{13}$$

$$I_7=\int_0^t T_1(\tau)r_1(\tau)\exp\left[-\frac{r_1^2(\tau)}{a(t-\tau)}\right]\frac{d\tau}{(t-\tau)^{3/2}}. \tag{14}$$

The function $r_1(t)$ in (8)-(14) is determined by the expression

$$r_1(t)=R_0-v_0\int_0^t \exp\left[-\frac{L\mu}{RT_1(\tau)}\right]d\tau. \tag{15}$$

Computations were performed on an "Odra-1304" electronic computer for lead particles with an initial radius of $R_0 = 7\cdot10^{-3}$ cm. The plasma temperature was assumed to be 6000°K, and the coefficient α = 5 W/cm$^2\cdot$deg.

Because (1) is meaningless for $r < 0$, a time interval was chosen for which the final radius remained positive. The final result is obtained by the machine after nine iterations with a linear initial approximation for the interval of t partitioned into 50 spaces and a relative accuracy of 2%.

The results are presented in Fig. 1 and Table 1. As is seen from the figure, the nature of the time change in the particle radius [the function $r_1(t)$] is in good agreement with the results of the experimental paper [1].

The contribution of the integrals I in (7) is represented in the table. As should have been expected, the contribution of the integrals I_1, I_2, I_4, which contain the quantity v as a factor, is small in the specific problem considered, which is characterized by a low evaporation rate:

$$v=\left|\frac{dr_1}{dt}\right|\approx 0.05\ \text{cm/sec}\ll\frac{a}{R_0}\approx 80\ \text{cm/sec}.$$

Moreover, the inequality

$$\frac{r_1(t)}{\Delta\tau}\ll\frac{a}{r_1(t)}.$$

is always conserved in the problem considered; hence, the contribution of the integral I_7 can also be neglected.

Estimates made permit finding certain important asymptotics.

At the beginning of the process, i.e., for $t \ll R_0^2/a$, although $T_1 \ll T_p$ the magnitude of the integrals I_5 and I_6 can be neglected in comparison with the value of the integral I_3, as is seen from (12) and (13). The exponential in the braces in this latter integral is much less than one; therefore, $T_1(t) \approx \sqrt{(a/\pi)} \cdot (\alpha T_p/\lambda)\sqrt{t}$. This result is evident from physical considerations since heating of a sphere with a practically fixed boundary occurs at the beginning of the process.

As is seen from the table, the integral I_3 continues to introduce the greatest contribution to (7) in the mode of the developed evaporation process, where its value is approximately constant in time. Solving the equation $dI_3/dt = 0$, we find

$$r_1^2(t) = A - Bt, \tag{16}$$

where A and B are certain constants. The Sreznevskii law follows from (16):

$$\frac{dS}{dt} \sim r_1 \frac{dr_1}{dt} = \text{const},$$

which is verified in [1]. Finally, for low values of r_1 the role of the integral I_6 grows,

$$T_1(t) \approx \frac{1}{2\sqrt{\pi a}} I_6, \tag{17}$$

for $r_1 \ll (\lambda/\alpha) \cdot (T_1/T_p)$.

In the specific problem under consideration, the inequality (17) starts to be satisfied from the value $r_1 \approx 10^{-3}$ cm. As is seen from the figure, the temperature of the evaporation surface hence varies weakly, and it can be extracted from under the integral sign in I_6. Solving the equation $dT_1/dt = 0$, we obtain

$$dr_1/dt = -C, \quad dS/dt \sim -Cr_1,$$

where C is some constant, i.e., for small values of the particle radius r_1 the rate of change of the surface area diminishes with time in proportion to the radius. This deviation from the Sreznevskii law is also detected in the experiments in [1].

NOTATION

R_0, initial particle radius; $r_1(t)$, radius of the evaporating particle; $T(r, t)$, temperature within the particle; $T_1(t)$, temperature of the evaporation surface; $G(r, \rho, t, \tau)$, Green's function; a, coefficient of thermal diffusivity; λ, coefficient of thermal conductivity; γ, metal density; μ, atomic weight; α, coefficient of heat exchange with the plasma; L, specific heat of evaporation; v_0, maximum evaporation rate; $q(t)$, heat flux from the plasma.

LITERATURE CITED

1. M. A. Luzhnova and Ya. D. Raikhbaum, Teplofiz. Vys. Temp., 7, No. 2 (1969).
2. É. A. Arinshtein, Izv. Vyssh. Uchebn. Zaved. Fiz., No. 4 (1961).
3. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, 2nd ed., Oxford University Press (1959).
4. J. P. Hirth and G. M. Pound, Evaporation and Condensation [Russian translation], Metallurgiya (1966).
5. S. I. Anisimov, Ya. A. Imas, G. S. Romanov, and Yu. V. Khodyko, Effect of High-Power Radiation on a Metal [in Russian], Nauka, Moscow (1970).